CENTRE FOR QUANTUM TECHNOLOGY, SINGAPORE

CLARENDON LABORATORY AND KEBLE COLLEGE, UNIVERSITY OF OXFORD

INSTITUTE FOR SCIENTIFIC INTERCHANGE, TURIN

Community detection in quantum networks

Tomi Johnson

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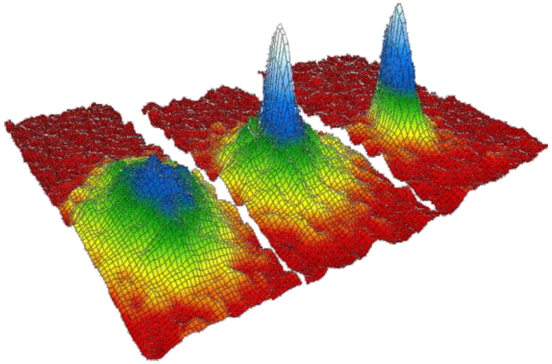
Quantum systems

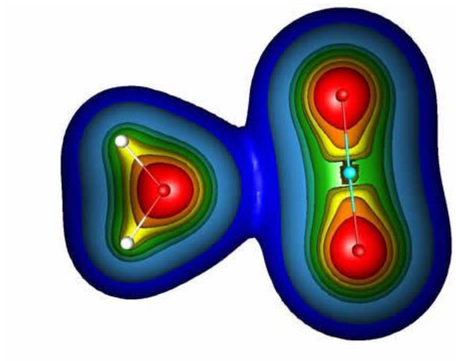
Atomic gas

(no structure)



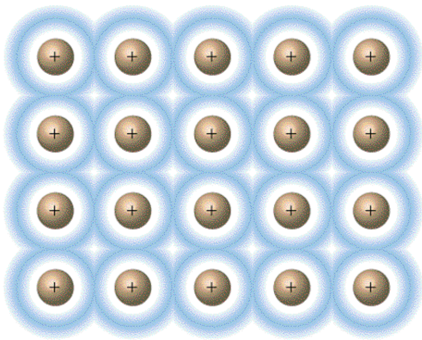
Few atoms

(structure tractable)

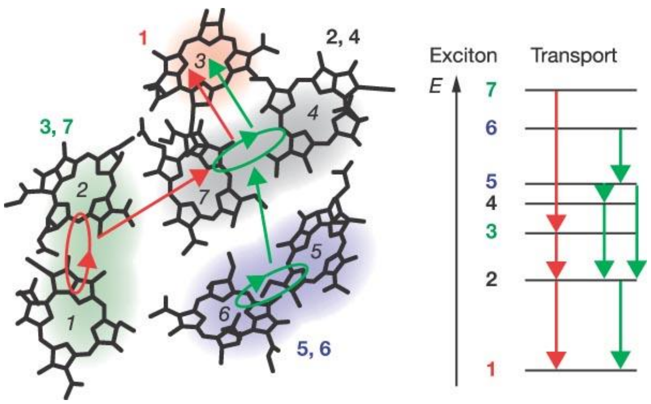


Solid state

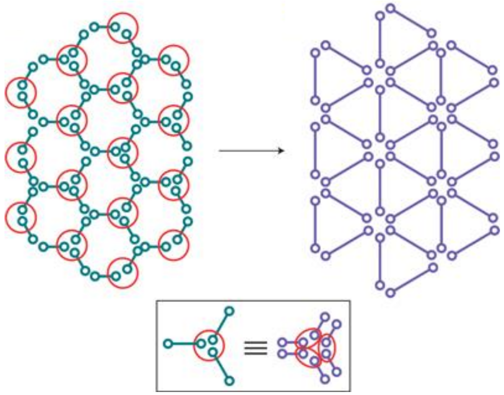
(periodic)



Exciton transport



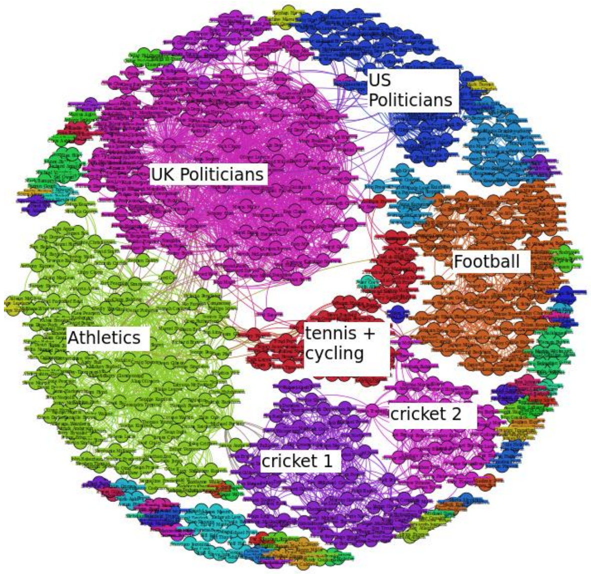
Quantum communication



Quantum/classical differences – Use

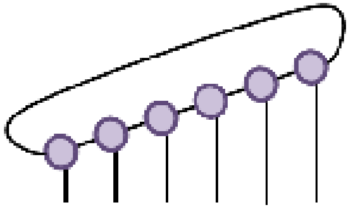
Classical:

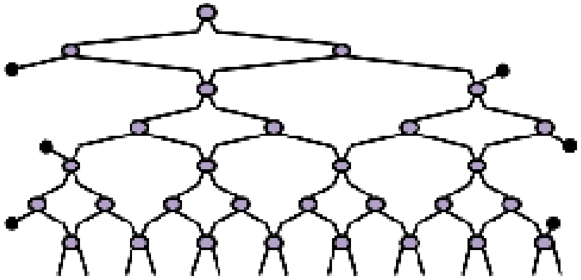
- Analysis
- Categorising

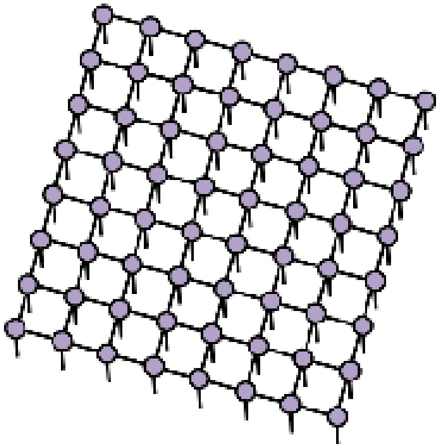


Quantum:

- Simulations
- Design







Quantum/classical differences – Expectations

What we can learn from classical:

- Approaches
- Difficulties

What we shouldn't transfer over:

- Expectations of size
- Expectations of efficiency
- Particular measures

What this work is not

- A competitor to classical community detection
- The only possible approach
- Targeted at only a single end goal
- The final word

The task

The nodes of our network are some orthonormal states

$$\{|i\rangle\}$$

that span the Hilbert space $\mathcal{H}$.

We seek to divide up the Hilbert space

$$\mathcal{H} = \bigoplus_{\mathcal{A} \in X} \mathcal{V}_{\mathcal{A}} \qquad \mathcal{V}_{\mathcal{A}} = \mathrm{span}_{i \in \mathcal{A}} \{|i\rangle\}$$

according to a Hamiltonian H .

Closeness, agglomeration, modularity

Closeness

$$c(\mathcal{A}, \mathcal{B})$$

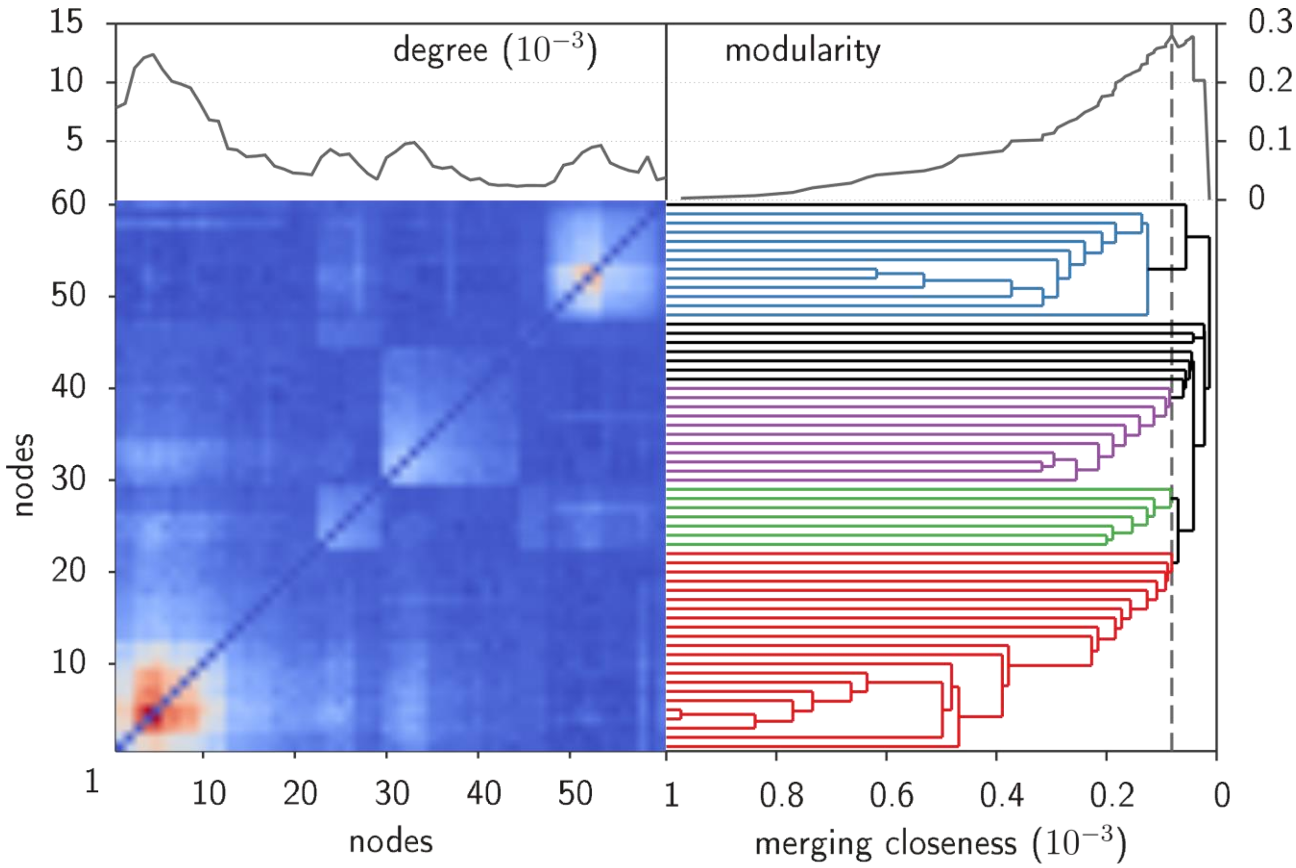
Modularity

$$Q(A)$$

Closeness (again)

$$A_{ij} = c(i, j)$$

$$i \neq j$$



Closeness in terms of physical quantity

Decide on a physical quantity we want to be small

$$Z_X = \sum_{\mathcal{A} \in X} Z_{\mathcal{A}}$$

Closeness relates to decrements of this quantity

$$c^Z(\mathcal{A}, \mathcal{B}) = \frac{Z_{\mathcal{A}} + Z_{\mathcal{B}} - Z_{\mathcal{A} \cup \mathcal{B}}}{|\mathcal{A}||\mathcal{B}|}$$

Transport and fidelity

Transport

$$T_{\mathcal{A}} = \frac{1}{2} \left| p_{\mathcal{A}} \left\{ \rho(t) \right\} - p_{\mathcal{A}} \left\{ \rho(0) \right\} \right|$$

$$c_0^T(i,j) = |H_{ij}|^2$$

$$c_\infty^T(i,j) = \sum_k | \langle i | \Lambda_k | j \rangle |^2$$

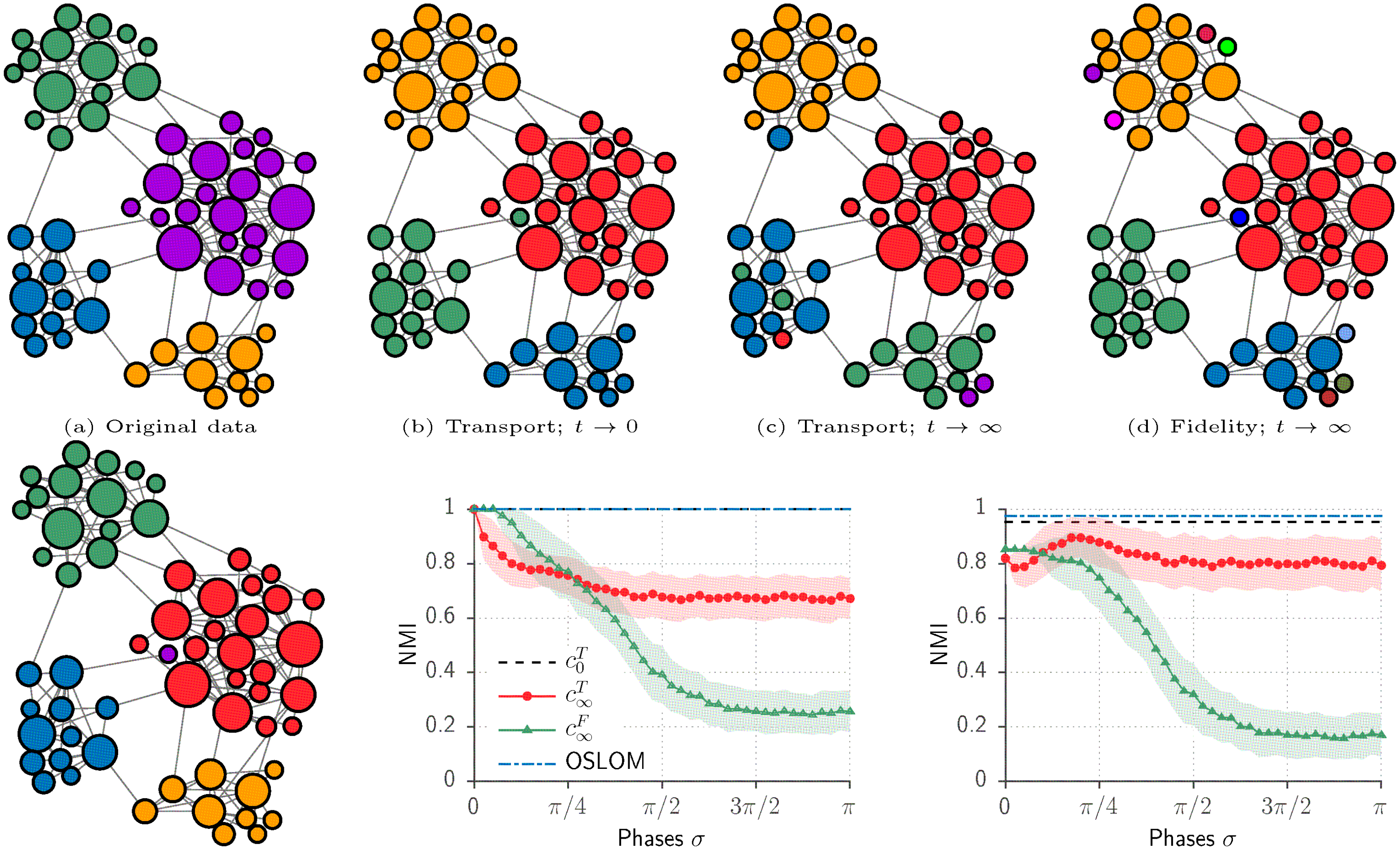
Fidelity

$$F_{\mathcal{A}}(t) = F^2 \left\{ \Pi_{\mathcal{A}} \rho(t) \Pi_{\mathcal{A}}, \Pi_{\mathcal{A}} \rho(0) \Pi_{\mathcal{A}} \right\}$$

$$c_\infty^F(i,j) = \sum_k \Lambda_k \rho_{ij}(0) \Lambda_k$$

First example

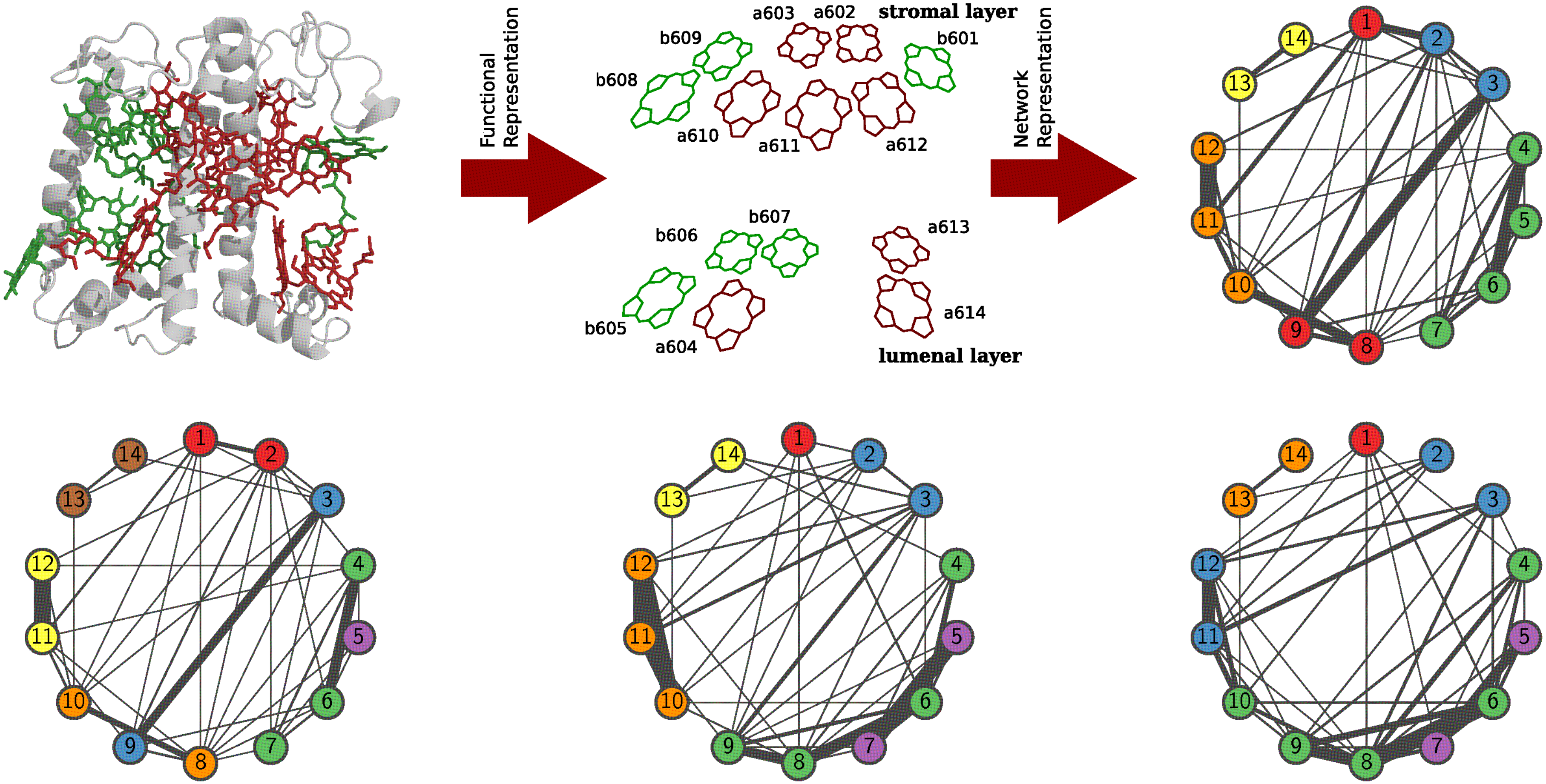
Quantum network from a classical network: $H_{ij} = A_{ij}$



(a) Original data (b) Transport; $t \to 0$ (c) Transport; $t \to \infty$ (d) Fidelity; $t \to \infty$

(e) OSLOM (f) Phases dependence (original partitioning) (g) Phases dependence (classical model)

Second example



(a) Transport; $t \to 0$ (b) Transport; $t \to \infty$ (c) Fidelity; $t \to \infty$

Summary and future directions

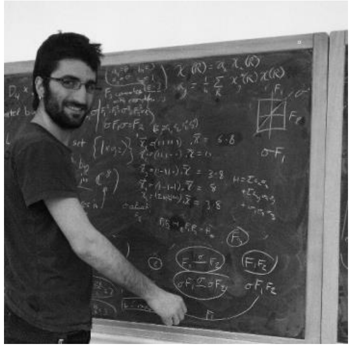
Communities should become important in quantum networks

Classical methods are not entirely appropriate

We have considered some possible methods

More possibilities, to be decided by specific applications

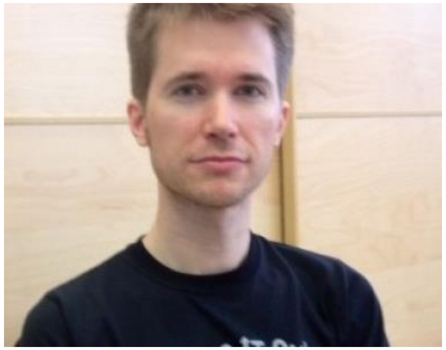
Thanks for listening



Mauro Faccin

Piotr Migdał

Jacob Biamonte



Ville Bergholm

M. Faccin, P. Migdał, T. H. Johnson, J. Biamonte, and V. Bergholm
Community Detection in Quantum Complex Networks
arXiv:1310.6638

Questions?