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Connecting quantum computation, topology and the efficiency of solving search problems using tensor networks

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Context

Network + transition rates = classical walks

Network + hopping amplitudes (+ noise) = quantum walks

Network + tensors = quantum computation
= search/counting problems

Layout

1. Reduction: (i) solving search and (ii) counting problems, (iii) simulating quantum computations to (iv) contracting tensor networks

[T. H. Johnson *et al.*, Sci. Rep. **3**, 1235 (2013)]
[Goldreich, *NP, and NP-completeness: The basics of computational complexity*, (CUP, 2010)]
[M. Van den Nest, Quant. Inf. Comp. **10**, 0258 (2010)]
[I. Arad and Z. Landau, SIAM J. Comput. **39**, 3089 (2010)]
[A. Garcia-Saez and J. I. Latorre, Quant. Inf. Comput, **12**, 0283 (2012)]

2. Algebraic and geometric conditions for efficiency

[Using many known circuit simulation and related algorithms]

3. Implications for the power of quantum computations

Reduction

Searching

↓

Counting

↓

Strong simulation

↓

Tensor network contraction

(In)efficiently solvable problems

Let x **describe an instance of a problem**

Problem is efficiently solvable iff a solution can be found in time $\leq p(|x|)$

[Goldreich, *NP, and NP-completeness: The basics of computational complexity*, (CUP, 2010)]

Search problem

Find some $w \in S(x)$ **where** $S(x)$ **is the set of solutions to** x

For a given w **it can be efficiently checked whether** $w \in S(x)$

[Goldreich, *NP, and NP-completeness: The basics of computational complexity*, (CUP, 2010)]

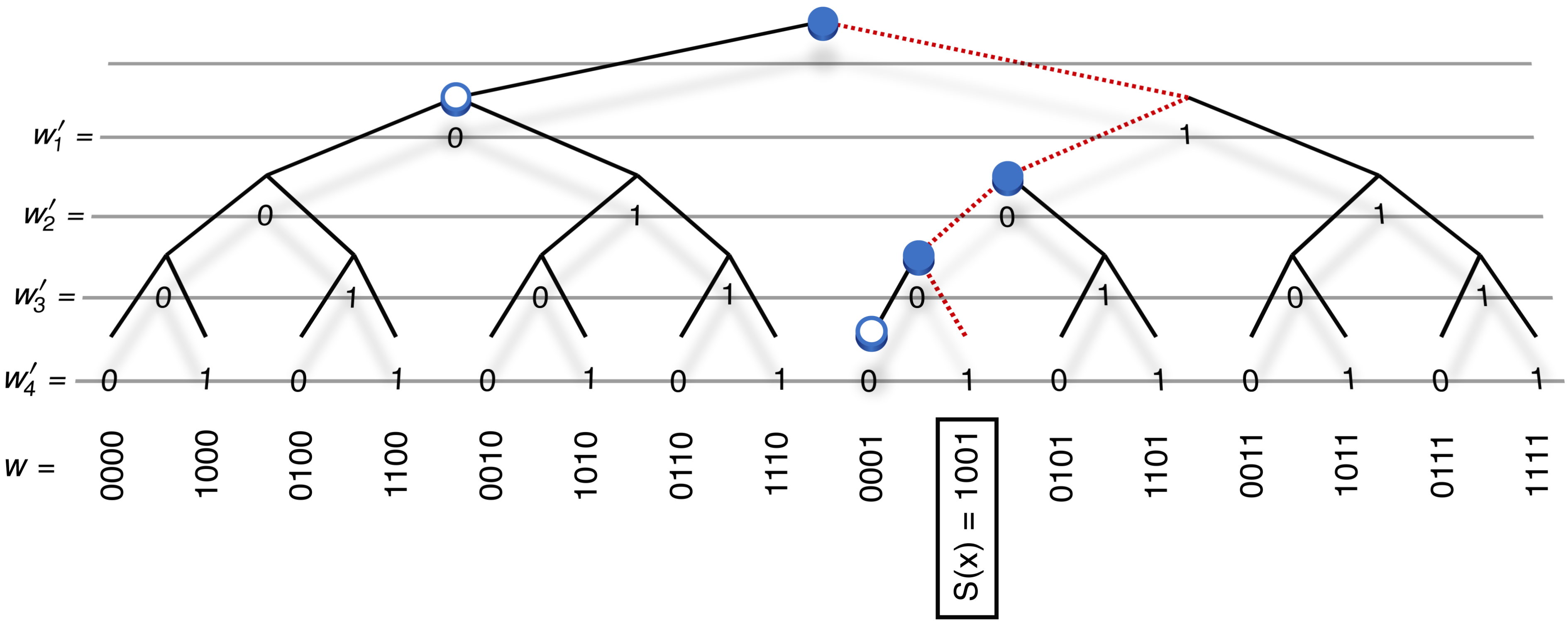
Counting problem

Instance: (x, n, w')

Find $\#(x, n, w')$ **the number of solutions to** x **of length** n **and suffix** w'

[Goldreich, *NP, and NP-completeness: The basics of computational complexity*, (CUP, 2010)]

Searching by counting



Solving $2|w|_{\min}$ **counting instances** (x, n, w') **solves** x

If the counting problem is efficiently solvable, so is the search problem

Counting by strongly simulating

Input $|w\rangle = |0\rangle^{\otimes(N-n)} |w_n\rangle \cdots |w_1\rangle$

Circuit $C_{x,n}$

Measurement $\{\hat{\Pi}, \mathbb{1} - \hat{\Pi}\}$ $\{\mathtt{yes}, \mathtt{no}\}$ $\hat{\Pi} = \mathbb{1}^{\otimes(N-1)} \otimes |1\rangle\langle 1|$

Solution checker for $|w\rangle$ then if $w \in S(x)$ then $\mathtt{yes}$ else $\mathtt{no}$

Counting by strongly simulating (2)

Solution counter for $|W(n,w')\rangle = |0\rangle^{\otimes(N-n)}|+\rangle^{\otimes(n-n')}|w'_{n'}\rangle\cdots|w'_1\rangle$

then `yes` is obtained with probability

$$\mathcal{P} = \mathcal{N}^2 \#(x,n,w')$$

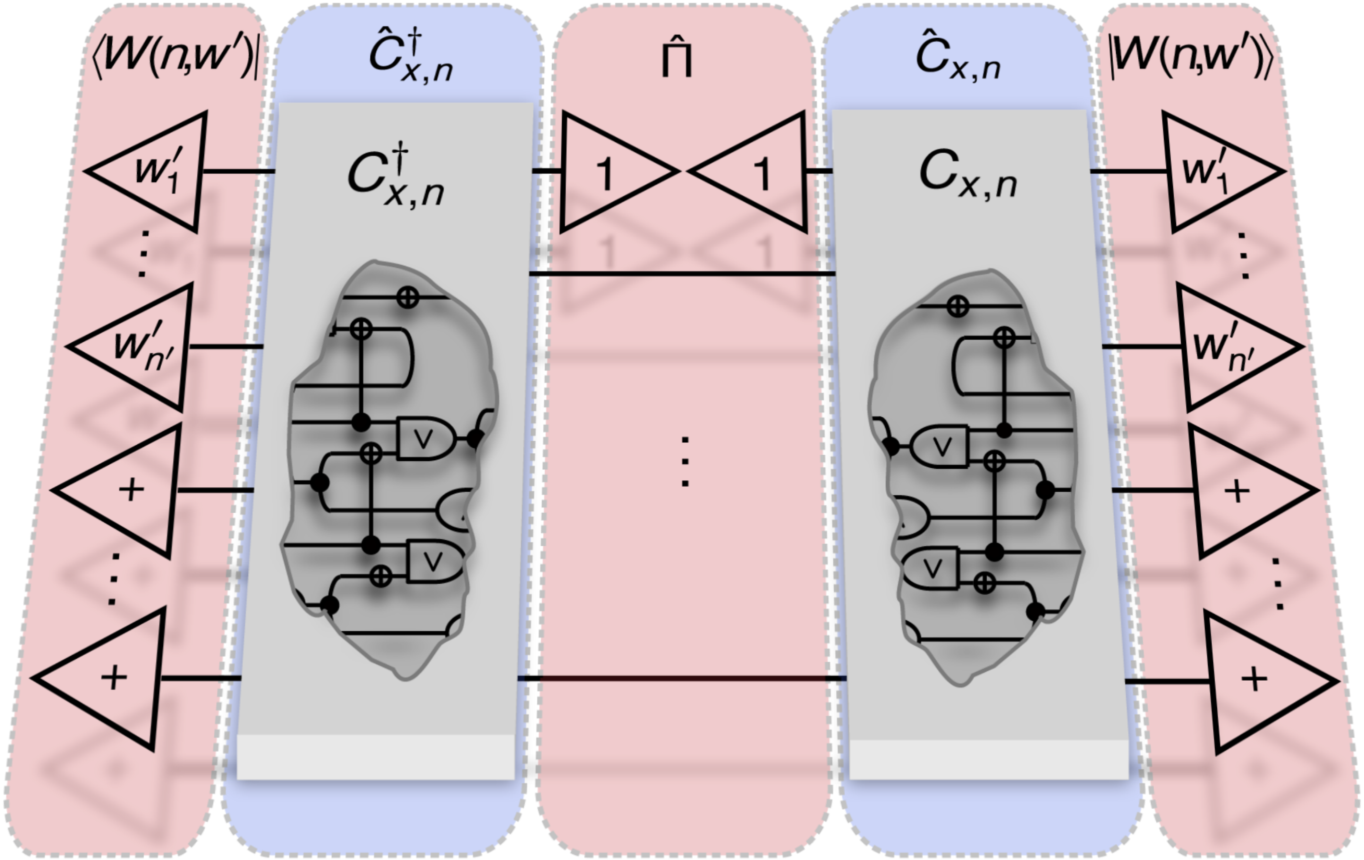
where $\mathcal{N} = 2^{(n'-n)/2}$ and $n' = |w'|$

Not experimental, but perhaps computational?
[R. Brüschweiler, Phys. Rev. Lett. **85**, 4815 (2000)]

Strong simulation

Strong simulation by tensor network contraction

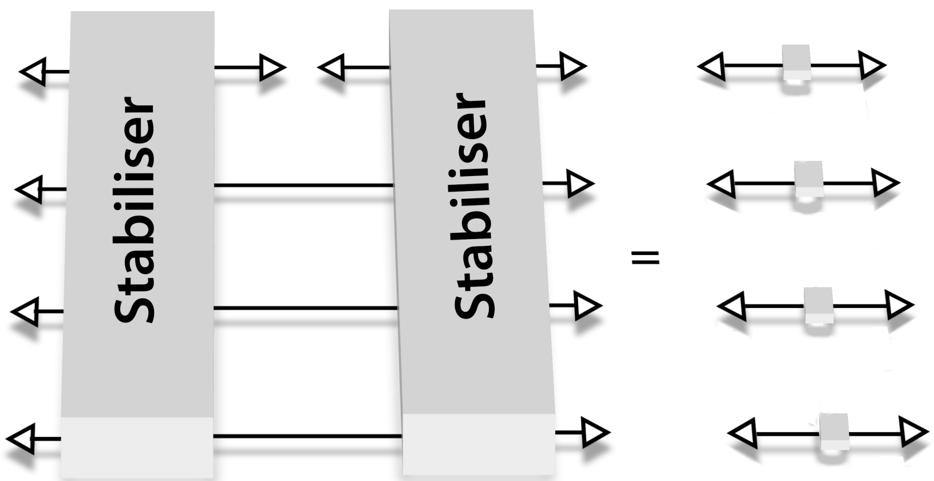
A visual representation of $\mathcal{P} = \langle W(n,w') \,|\, \hat{C}^{\dagger}_{x,n} \hat{\Pi} \hat{C}_{x,n} \,|\, W(n,w') \rangle$



(Efficiently) contractible networks

Stabiliser circuits

Clifford gates: H, CNOT, P

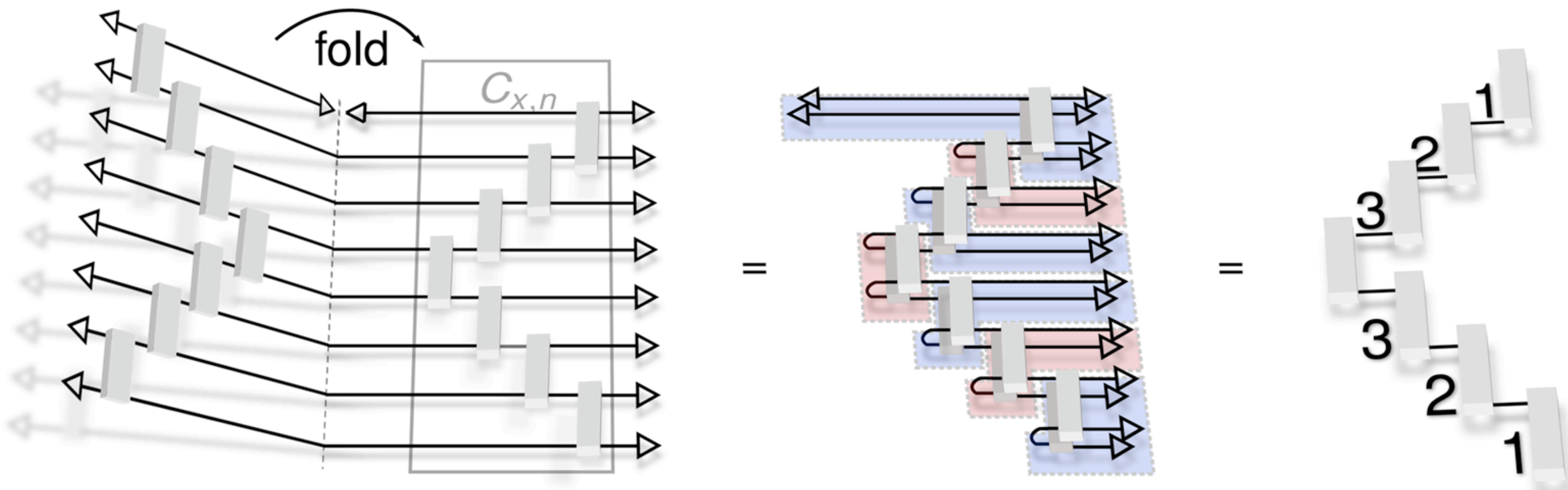


[D. Gottesman, Phys. Rev. A **57**, 127137 (1998)]
[S. Aaronson and D. Gottesman, Phys. Rev. A **70**, 052328 (2004)]
[S. Anders and H. J. Briegel, Phys. Rev. A **73**, 022334 (2006)]
[S. Clark, R. Jozsa, and N. Linden, Quant. Inf. Comp. **8**, 0106 (2008)]

(Efficiently) contractible networks (2)

Tree-like circuits

At most logarithmically increasing treewidth

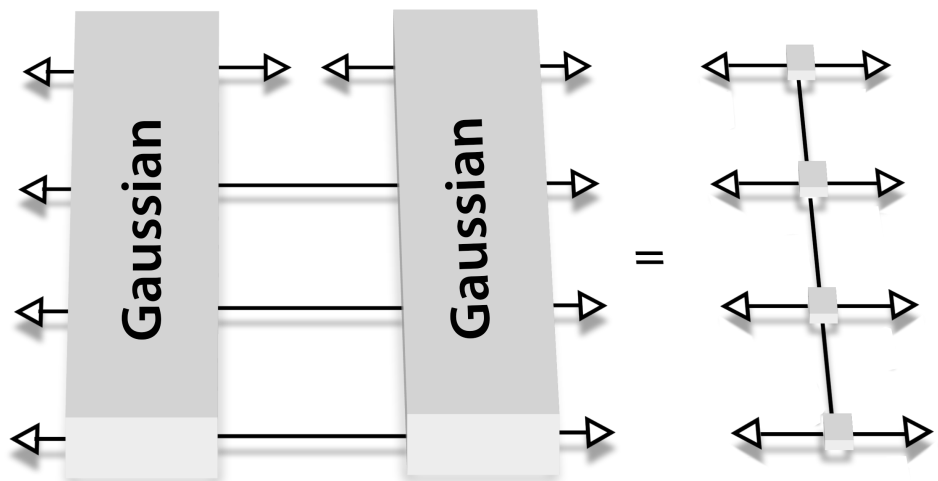


[I. L. Markov, Y.-Y. Shi, SIAM J. Comput. **38**, 963 (2008)]

(Efficiently) contractible networks (3)

Gaussian circuits (a.k.a. matchgate, free-fermion circuits)

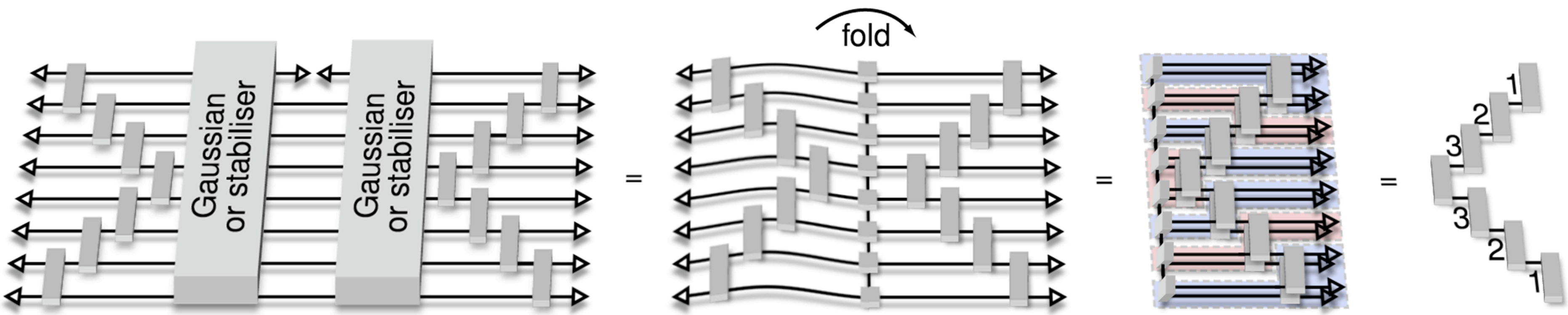
Gaussian operators $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$ $B = \begin{pmatrix} e & f \\ g & h \end{pmatrix}$ $G(A, B) = \begin{pmatrix} a & 0 & 0 & b \\ 0 & e & f & 0 \\ 0 & g & h & 0 \\ c & 0 & 0 & d \end{pmatrix}$



[L. G. Valiant, SIAM J. Comput. **31**, 1229 (2002)]
[E. Knill, preprint quant-ph/0108033 (2001)]
[B. M. Terhal and D. P. DiVincenzo, Phys. Rev. A **65**, 032325 (2002)]
[D. P. DiVincenzo, and B. M. Terhal, Found. Phys. **35**, 1967 (2005)]
[S. Bravyi, Quantum Inf. Comp. **5**, 216 (2005)]
[R. Jozsa and A. Miyake, Proc. R. Soc. A **464**, 3089 (2008)]

(Efficiently) contractible networks (4)

New type of simulable circuit



(Efficiently) contractible networks (5)

Matrix product state ansatz

[G. Vidal, Phys. Rev. Lett. **91**, 147902 (2003)]

[A. SaiToh and M. Kitagawa, Phys. Rev. A **73**, 062332 (2006)]

Matchgate tensor network

[S. Bravyi, Contemporary Mathematics **482**, 179 (2009)]

Implications

Limits to the speed up offered by quantum computations of simple geometries

Can demonstrate that a search problem is efficiently solvable by showing that its solutions can be checked by efficiently simulable circuits.

A new efficient strong simulation method leads to new efficiently solvable counting (and search) problems

Implications (2)

Efficiently strongly simulable circuits decide all problems in P?

We can efficiently solve all problems in #P and NP

[Goldreich (CUP, 2010)]

#P is only as difficult as P, P = NP

Initial assumption unlikely

Summary and outlook

Tensor networks are one way of using network concepts in quantum (or other) systems

Bound power of quantum computation

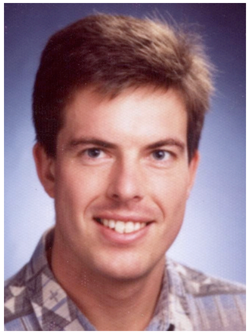
Provide methods for efficiently solving counting problems

Thanks for listening

Collaborators

Jacob Biamonte

Stephen Clark



Dieter Jaksch

Sci. Rep. **3**, 1235 (2013)
arXiv:1209.6010

Figures by Federica Ferarris

Questions?