Thermometry of arbitrary quantum systems via non-equilibrium work distributions:

Application to bosons in a lattice

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Equilibrium probe I

- System $\hat{H}_S$ in equilibrium $\hat{\rho}_S \propto \mathrm{e}^{-\beta \hat{H}_S}$ at inv. temperature β
- Interact with probe e.g. qubit $(\Delta/2)\hat{\sigma}_z$
- Let probe thermalise

$$\hat{\rho}_q \propto \hat{\mathbb{1}} - 2\tanh(\beta\Delta)\hat{\sigma}_z$$

- Measure probe $\langle\hat{\sigma}_z\rangle \propto \tanh(\beta\Delta)$
- Infer β

Equilibrium probe II

Good:

- Don't need to know $\hat{H}_S$
- Just need to know $(\Delta/2)\hat{\sigma}_z$

Bad:

- Need to precisely know and tune $\Delta \sim 1/\beta$
- Not easy at low temperatures

Non-equilibrium probe I

Popular method:

- QuProCS

For thermometry:

- Good: Not necessary that probe energy $\sim 1/\beta$
  - M. Bruderer and D. Jaksch, New J. Phys. **8**, 87 (2006)
  - C. Sabín, A. White, L. Hackermuller, and I. Fuentes, Sci. Rep. **4**, 6436 (2014)
  - D. Hangleiter, M. T. Mitchison, THJ, M. Bruderer, M. B. Plenio, and D. Jaksch, Phys. Rev. A **91**, 013611 (2015)
- Bad: Only applicable to a few simple systems

Non-equilibrium probe II

What we need:

- Generic temperature-dependence in non-equilibrium probe dynamics

Where do we get this:

- Relationship between non-equilibrium work-distributions

Non-equilibrium work distributions

Consider a quench Q:

- System Hamiltonian $\hat{H}(\lambda) = \hat{H}_S + \lambda \hat{V}$
- System begins in equilibrium
- Parameter λ changed over time τ
- Measure the work distribution $P_Q(W)$

Consider a forward and backwards quench

- F from λ_i to λ_f
- B from λ_f to λ_i

Crooks-Tasaki: P. Talkner and P. Hanggi, J. Phys. A **40**, F569-571 (2007)

$$\ln \left\{ \frac{P_F(W)}{P_B(-W)} \right\} = \beta(W - \Delta F)$$

with free energy difference $\Delta F = F_f - F_i$

Qubit interferometry I

How to measure work distribution $P_Q(W)$?

- R. Dorner, S. Clark, L. Heaney, R. Fazio, J. Goold, and V. Vedral, Phys. Rev. Lett. **110**, 230601 (2013).
- L. Mazzola, G. De Chiara, and M. Paternostro, Phys. Rev. Lett. **110**, 230602 (2013)

- Add qubit probe $\hat{H}_T(t) = (\Delta/2)\hat{\sigma}_z + \hat{H}_S + \hat{H}_I(t)$
- Each state interacts with different interaction strength

$$\hat{H}_I(t) = (g_\downarrow(t)|\downarrow\rangle\langle\downarrow| + g_\uparrow(t)|\uparrow\rangle\langle\uparrow|) \otimes \hat{V}$$

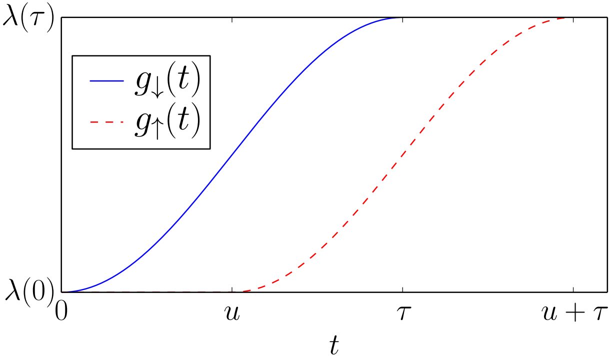
- Start at $t = 0$

$$\hat{\rho} = |+\rangle\langle+| \otimes \hat{\rho}_S(\lambda_Q(0))$$

with the qubit in some superposition $|+\rangle = (|\downarrow\rangle + |\uparrow\rangle)/\sqrt{2}$

Qubit interferometry II

- Quenched with a delay u



Qubit interferometry III

- Qubit ends up in state

$$\hat{\rho}_q = \frac{1}{2}\left(\hat{\mathbb{1}} + c_x\hat{\sigma}_x + \mathrm{i}c_y\hat{\sigma}_y\right)$$

$$c_x + \mathrm{i}c_y \propto \chi_Q(u)$$

  with decoherence (characteristic) function $\chi_Q(u)$
- Measure it

$$\langle\hat{\sigma}_x\rangle + \mathrm{i}\langle\hat{\sigma}_y\rangle \propto \chi_Q(u)$$

- Find work distribution

$$P_Q(W) = (2\pi)^{-1}\int \mathrm{d}u\mathrm{e}^{-\mathrm{i}Wu}\chi_Q(u)$$

Ideal thermometry

What don't we need:

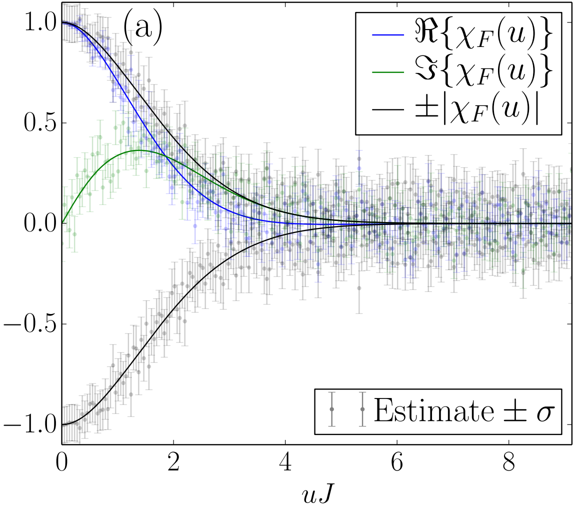
- System Hamiltonian $\hat{H}_S$
- Coupling $\hat{V}$
- Tuned qubit $\Delta \sim 1/\beta$

What do we need:

- Both qubit states couple to the same (potentially unknown) $\hat{V}$
- The quenches match

Inferring β I

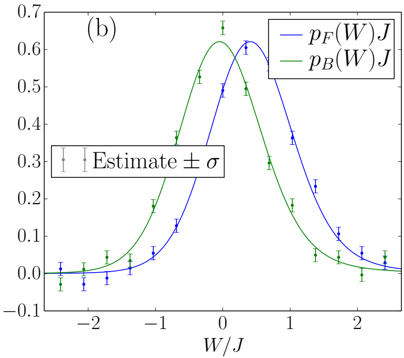
- Estimate $\chi_Q(u_j)$ using N_{meas} measurements, N_{steps} time point, up to T



Inferring β II

- Estimate

$$p_Q(W) = \frac{T}{2\pi N_{\text{steps}}} \left(1 + 2\Re \left\{ \sum_{j=1}^{N_{\text{steps}}} \mathrm{e}^{-\mathrm{i}Wu_j} \chi_Q(u_j) \right\} \right)$$

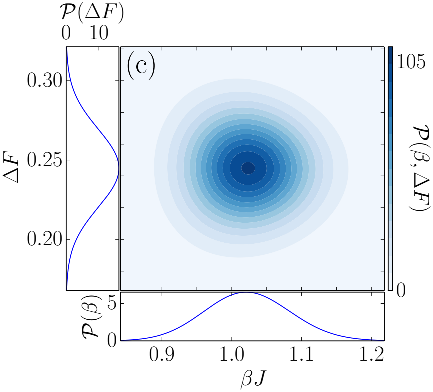


Inferring β III

- Use Crooks' relation $\ln\{p_F(W)/p_B(-W)\} = \beta(W - \Delta F)$ and Bayesian analysis

$$\mathcal{P}(\beta, \Delta F|O) = \frac{\mathcal{P}(O|\beta, \Delta F)\mathcal{P}(\beta, \Delta F)}{\int \mathrm{d}\beta \mathrm{d}(\Delta F)\mathcal{P}(O|\beta, \Delta F)\mathcal{P}(\beta, \Delta F)}$$

to classify state of knowledge given observations O



Example system: cold atoms, BHM

General:

- System — cold atomic gas
- Qubit — two internal states of an impurity atom of a different species, strongly localised
- Coupling — both states, $\hat{V} = \int \mathrm{d}\mathbf{r}\, n_q(\mathbf{r}) \hat{\Psi}^\dagger(\mathbf{r}) \hat{\Psi}(\mathbf{r})$

Specific:

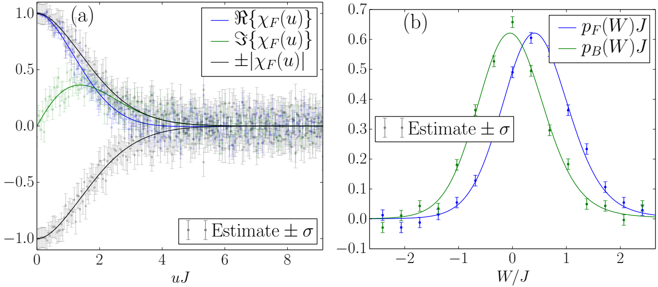
- Bose Hubbard model

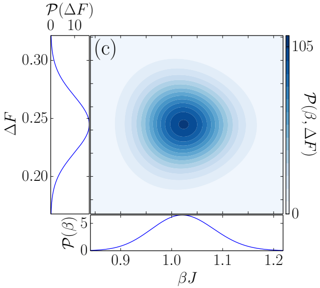
$$\hat{H}_S = -J \sum_{\langle jj' \rangle} \hat{a}_j^\dagger \hat{a}_{j'} + \sum_{j=1}^{M} \left(\frac{U}{2} \hat{a}_j^\dagger \hat{a}_j^\dagger \hat{a}_j \hat{a}_j - \mu \hat{a}_j^\dagger \hat{a}_j \right)$$

- On-site interaction

$$\hat{V} = \eta \hat{a}_c^\dagger \hat{a}_c$$

Results — superfluid I





$\beta J = 1$

$nU/J = 0.1$

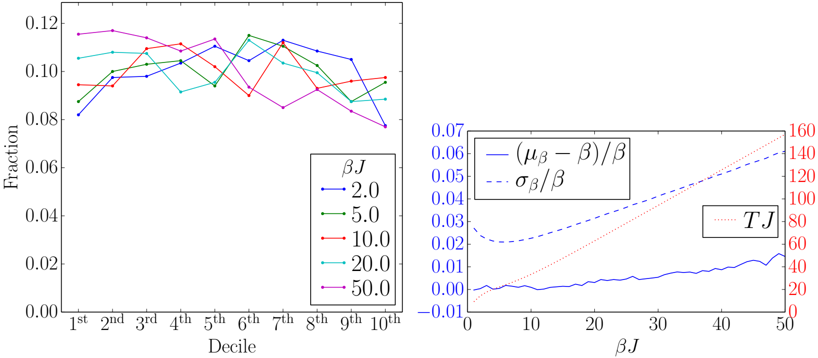
$\lambda_i\eta/J, \lambda_f\eta/J = 0, 0.5$

$\tau J, TJ = 1, 9$

$N_{\mathrm{meas}} = 500$

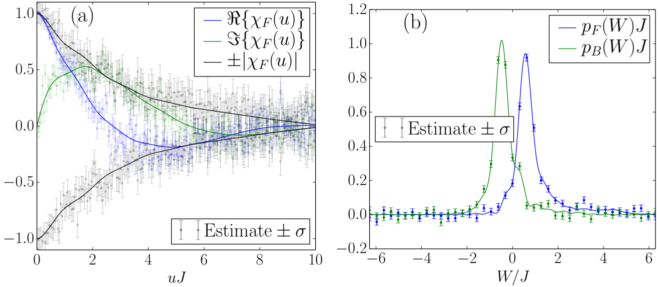
$N_{\mathrm{steps}} = 200$

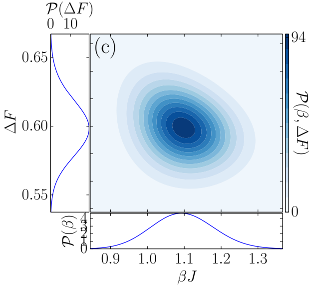
Results — superfluid II



Params as before, except $N_{\text{steps}} = 1000$.

Results — stronger interactions





$\beta J = 1$

$nU/J = 4$

$\lambda_i\eta/J, \lambda_f\eta/J = 0, 2$

$\tau J, TJ = 0.1, 10$

$N_{\mathrm{meas}} = 500$

$N_{\mathrm{steps}} = 200$

Summary and conclusions

Key points:

- Low-temperature
- Generic
- Works for mildly-interacting Bose gas

Ideas:

- Distinguish thermal from non-thermal?

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Questions?

Thanks for listening